A new goodness-of-fit test for time series models based on correlation between the sample autocorrelation and partial autocorrelation sequences

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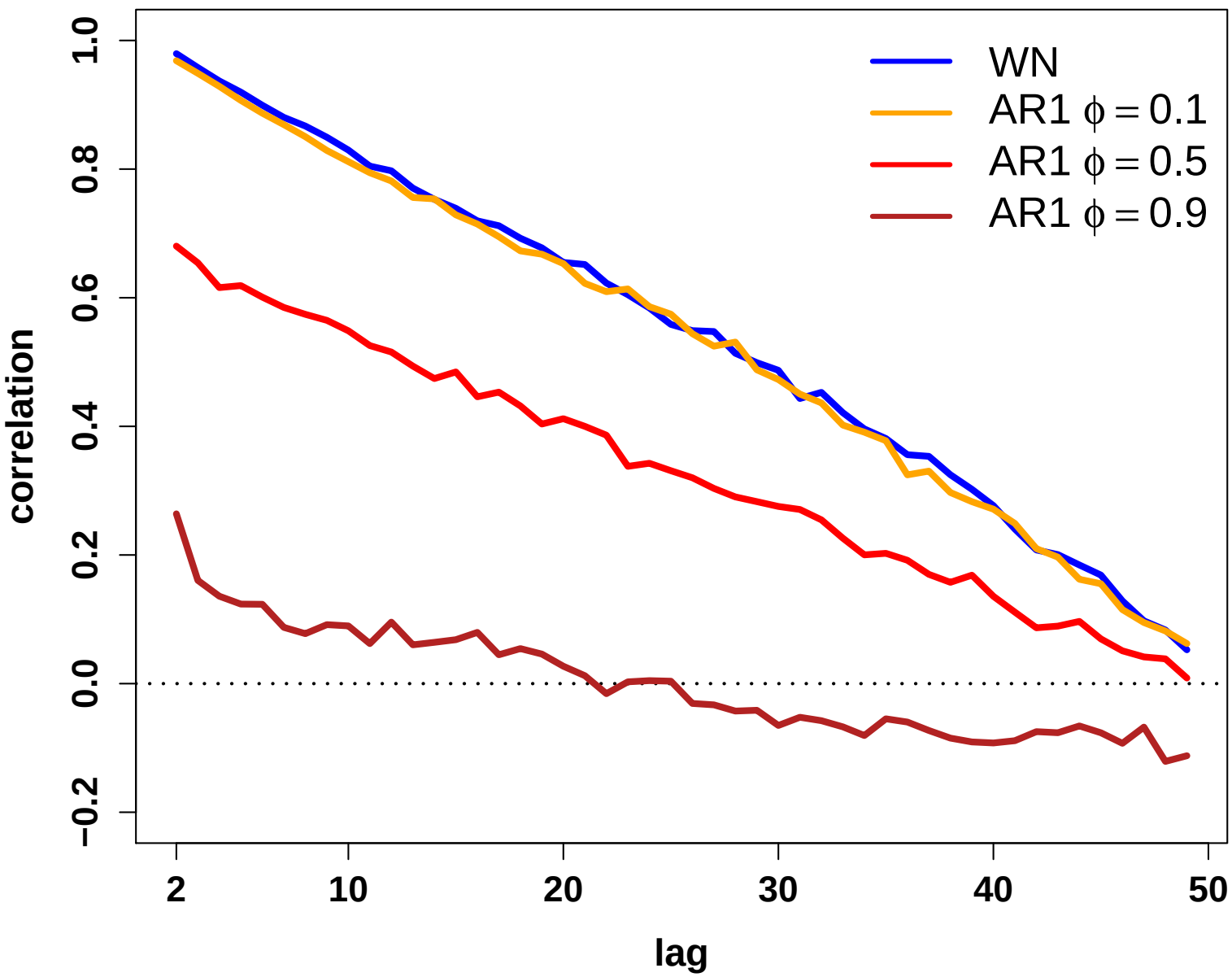
Objectives

1. Develop a test for white noise using correlation between sample ACF and PACF
2. Compare power of new test to several other portmanteau tests


Correlation between ACF and PACF at each lag: WN vs. AR(1)
WN
AR1 ϕ = 0.1
AR1 ϕ = 0.5
AR1 ϕ = 0.9
correlation
1.0
0.8
0.6
0.4
0.2
0.0
−0.2
2
10
20
30
40
50
lag

Expected Correlation Between ACF and PACF

- Correlation at lag h

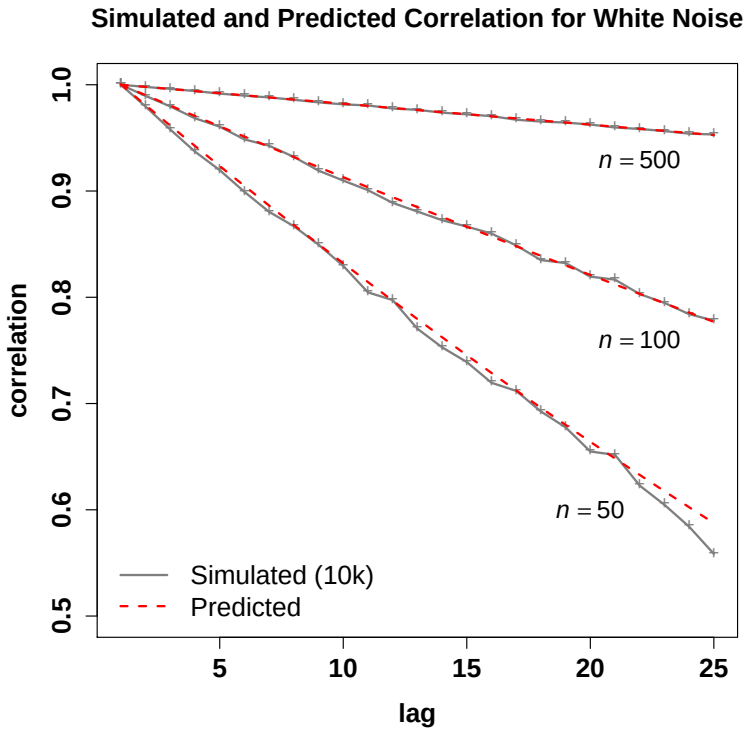
$$\text{Cor}(\hat{\rho}(h), \hat{\pi}(h)) \approx$$

$$\frac{4n(n-h+1)+h(h+2)}{4n^2}$$

- Variance of Difference

$$\text{Var}(\hat{\rho}(h) - \hat{\pi}(h)) \approx$$

$$\frac{(n-h)[4n(h-1)+h(h+2)]}{2n^3(n+2)}$$



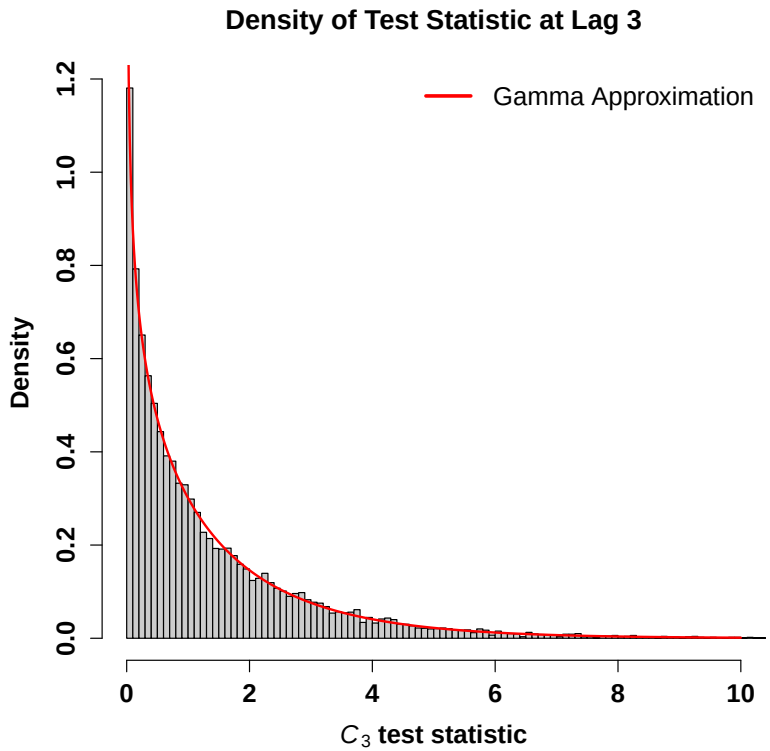
Test Statistic

- Test Statistic

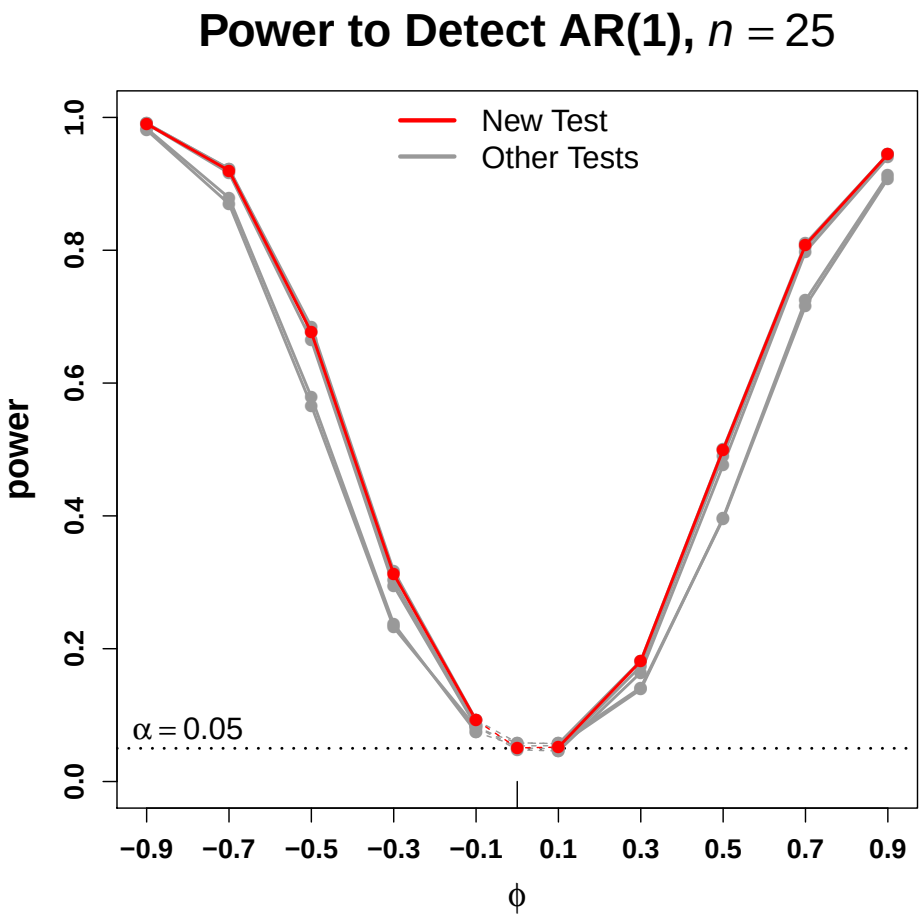
$$C_m = \sum_{h=2}^{m} \frac{|\hat{\rho}(h) - \hat{\pi}(h)|}{\sqrt{\text{Var}(\hat{\rho}(h) - \hat{\pi}(h))}}$$

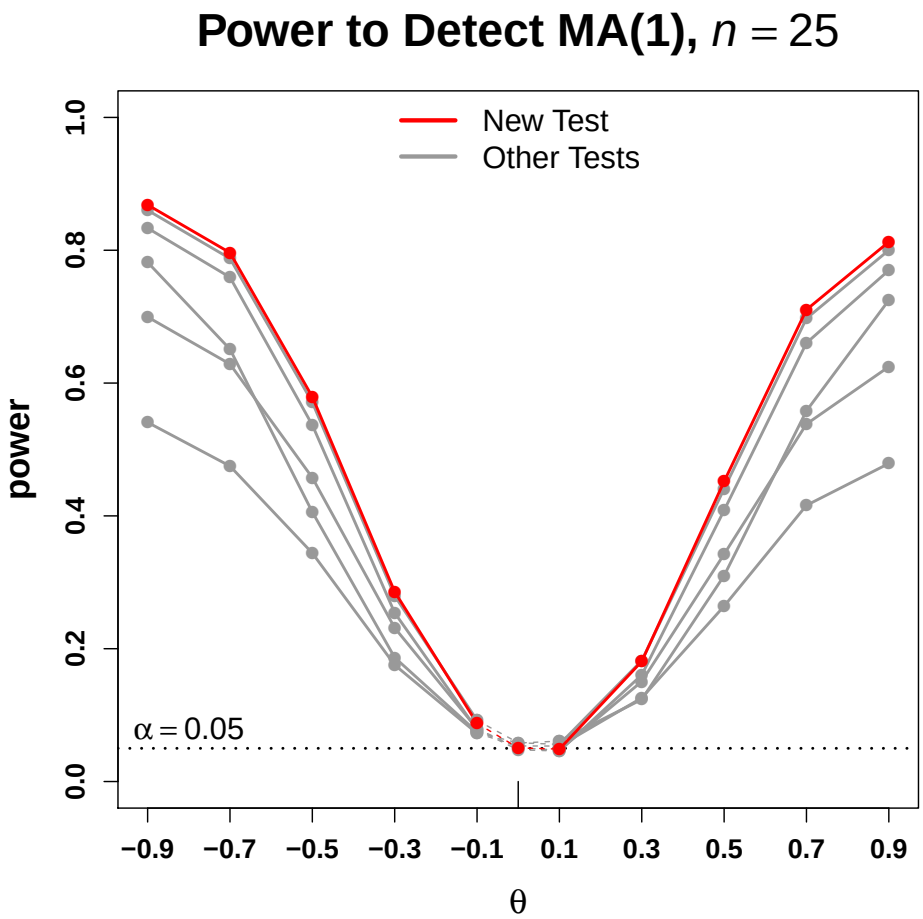
- Distribution

$$C_3 \sim \text{Gamma}\left(\frac{3}{4}, \frac{1}{2}\right)$$



Power for Testing AR(1) and MA(1)





Power for Testing Fractional ARIMA Processes