Introduction to nonstationarity

Spring 2017 Spatiotemporal Reading Group

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April 18, 2017

Types of stationarity

For a spatial process $Y(\boldsymbol{s}), \boldsymbol{s} \in D$.

- Strong stationarity

$$\Pr(Y(\boldsymbol{s}_1 + h), Y(\boldsymbol{s}_2 + h), \ldots) = \Pr(Y(\boldsymbol{s}_1), Y(\boldsymbol{s}_2), \ldots)$$

- Weak (second-order) stationarity

$$\mathbb{E}[Y(\boldsymbol{s})] = \mu$$

and

$$\mathrm{Cov}(Y(\boldsymbol{s}_1), Y(\boldsymbol{s}_2)) = \mathrm{C}(\boldsymbol{s}_1 - \boldsymbol{s}_2)$$

Nonstationarity in covariance

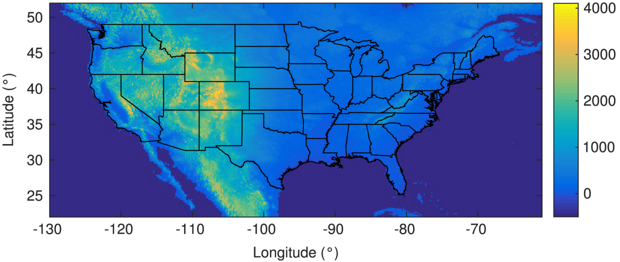

Figure 1: Fulgstad et al. 2015

Why is this hard?

- More parameters
- Trend-covariance tradeoff
- Limited data

Approaches for fitting

- Smoothing and kernel-based methods
- Basis function models
- Process convolution models
- Spatial deformation models

Smoothing and Kernel-based methods

- Consider local areas stationary; divide into subregions
- Estimate parameters locally
- Weight kernels by distance from subregion center

$$Y(\boldsymbol{s}) = \sum_{i=1}^{k} w_i(\boldsymbol{s}) Y_i(\boldsymbol{s})$$

Can be extended to

$$Y(\boldsymbol{x}) = \int_D w(\boldsymbol{x} - \boldsymbol{s}) Y_{\theta(\boldsymbol{s})}(\boldsymbol{x}) \, d\boldsymbol{s}$$

Basis function models

Spectral decomposition of empirical covariance matrix

$$\hat{\boldsymbol{\Sigma}}_Y \boldsymbol{\Phi} = \boldsymbol{\Phi} \boldsymbol{\Lambda}$$

- Requires multiple realizations

Fourier of Karhunen-Loeve expansions

- Latent spatial power process induces nonstationarity

Wavelet models

Process convolution models

$$Y(\boldsymbol{s}) = \int_{\mathbb{R}^2} k(\boldsymbol{s} - \boldsymbol{u}) \zeta(\boldsymbol{u}) \, du$$

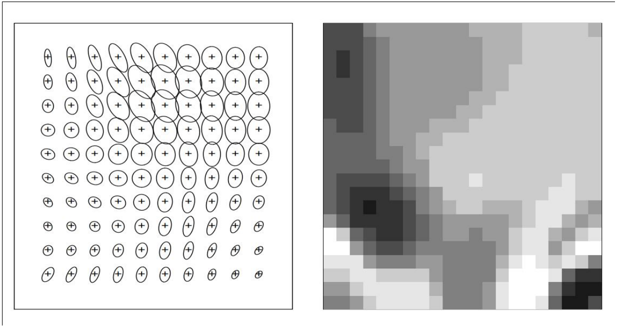

Figure 2: Calder & Cressie 2007

Spatial deformation model

Need temporal replicates;

$$Y(\boldsymbol{x}, t) = \mu(\boldsymbol{x}, t) + \nu(\boldsymbol{x})^{1/2} E_t(\boldsymbol{x}) + E_\epsilon(\boldsymbol{x}, t)$$

- $\mu(\boldsymbol{x}, t)$: mean field
- $\nu(\boldsymbol{x})$: smooth function, spatial variance
- $E_t(\boldsymbol{x})$: standard second-order continuous Gaussian process
- $E_\epsilon(\boldsymbol{x}, t)$: measurement error/short scale structure

$$\mathrm{Cor}(E_t(\boldsymbol{x}), E_t(\boldsymbol{y})) = \rho_\theta(\|f(\boldsymbol{x}) - f(\boldsymbol{y})\|)$$

Discussion

- Effects of uncertainty in spatial structure
- Understanding and calibrating Bayesian priors
- Model diagnostics
- Software/accessibility